MATH141 Summer I
Exam 1 [100 pt]

Instructions: Number the answer sheets from 1 to 4 and fill out all the information in each of them (sign the Honor Pledge on page 1 only). Solve only one problem in every answer sheet. If you need more space to solve a given problem, use the back of the same answer sheet. No lecture notes, cheat sheets, books, or electronic devices of any kind are allowed.

1. [20 pt]

   (a) Let R be the region enclosed by the graphs of $f(x) = x\sin(x^3)$ and $g(x) = 0$ for $0 \leq x \leq \sqrt[3]{\pi}$. Calculate the volume of the solid obtained by revolving the region R about the y-axis.

   (b) Let R be the region enclosed by the graphs of $f(x) = x+2$ and $g(x) = x^2$. Calculate the volume of the solid obtained by revolving the region R about the x-axis.

2. [20 pt] Let $f(x) = x^2 - \frac{1}{8}\ln x$ for $1 \leq x \leq e$. Find the length of the graph of f.

3. [20 pt] Find the center of gravity of the quarter of a circle enclosed by the graphs of $f(x) = \sqrt{4-x^2}$, $g(x) = 0$ and $x = 0$.

   You may use the fact that the area of a circle of radius r is equal to πr^2.

   Justify any symmetry argument analytically (with formulas). In particular, write the formulas for the moments and the area of the region.

4. [20 pt] A tank has the shape of an inverted cone with depth equal to 2 ft and diameter equal to 4 ft at the top of the tank. The tank is initially filled with a liquid weighing 100 lb/ft^3.

   Calculate the amount of work required to pump all the liquid to a level 2 feet above the top of the tank.

5. [20 pt] The equations $x = 3\cos^3 t$, $y = 3\sin^3 t$ for $0 \leq t \leq 2\pi$ parametrize an **astroid**.

   (a) Calculate the length of the segment of this astroid for $0 \leq t \leq \frac{\pi}{2}$.

   (b) Calculate the length of the curve for $0 \leq t \leq 2\pi$. **For this second part only: keep in mind that** $\sqrt{y^2} = |y|$.