

## MATH141 Fall 2019

## Exam 2 [100 pt]

**Instructions:** Number the answer sheets from 1 to 4 and fill out all the information in each of them (sign the Honor Pledge on page 1 only). Solve only one problem in every answer sheet. If you need more space to solve a given problem, use the back of the same answer sheet. No lecture notes, cheat sheets, books, or electronic devices of any kind are allowed.

1. [20 pt]

(a) Let  $f(x) = \frac{x}{9+x^2}$ . Find the largest interval containing  $x = 1$  for which  $f$  has an inverse. Find  $(f^{-1})'(\frac{1}{10})$ .

(b) Let  $g(x) = x^{\cos x}$ . Find the largest possible domain  $g$  can have and calculate  $g'(x)$ .

2. [20 pt] Calculate the following integrals

(a)  $\int \frac{dx}{\sin^{-1} x \sqrt{1-x^2}}$

(b)  $\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x} dx$

3. [20 pt] Calculate the following integrals

(a)  $\int \frac{\ln y}{\sqrt{y}} dy$

(b)  $\int_0^{\frac{\pi}{2}} (x^2 + 1) \sin x dx$

4. [20 pt] Calculate the following limits

(a)  $\lim_{x \rightarrow 0^+} \frac{\sin(3x)}{\sin^{-1}(2x)}$

(b)  $\lim_{x \rightarrow \infty} x^{\frac{1}{x}}$

(c)  $\lim_{x \rightarrow \infty} \frac{\ln(x + e^{2x})}{x}$

5. [20 pt] Solve the following differential equations

(a)  $\frac{dy}{dx} + e^{x+y} = 0, \quad y(1) = 1$

(b)  $\frac{dy}{dx} + \frac{1}{1+x}y = (1+x), \quad x > 0, \quad y(1) = 1$