MATH141 Fall 2019
Exam 3 [100 pt]

Instructions: Number the answer sheets from 1 to 4 and fill out all the information in each of them (sign the Honor Pledge on page 1 only). Solve only one problem in every answer sheet. If you need more space to solve a given problem, use the back of the same answer sheet. No lecture notes, cheat sheets, books, or electronic devices of any kind are allowed.

1. [20 pt] Calculate the following integrals

    (a) $\int_0^{\pi/2} \sin^2 x \cos^2 x \, dx$

    (b) $\int \tan^5 x \sec^4 x \, dx$

2. [20 pt] Calculate the following integrals

    (a) $\int \dfrac{x}{\sqrt{9-x^2}} \, dx$

    (b) $\int_{\sqrt{2}}^2 \dfrac{1}{x^2\sqrt{x^2-1}} \, dx$

3. [20 pt] Calculate the following integrals

    (a) $\int_0^1 \dfrac{x-1}{x^2+3x+2} \, dx$

    (b) $\int \dfrac{1}{x(x-1)^2} \, dx$

4. [20 pt] Determine if the following integrals converge. If they do, find their values.

    (a) $\int_0^\infty \dfrac{x}{(x^2+2)^2} \, dx$

    (b) $\int_1^2 \dfrac{1}{x(\ln x)^{1/2}} \, dx$

5. [20 pt]

    (a) Fill in the blanks to give an approximation to $\int_2^4 \frac{1}{x} dx$ using the trapezoidal rule with $n=4$.

    $$\int_2^4 \frac{1}{x} dx \approx \frac{\quad}{\quad} \left[\frac{\quad}{\quad} + \quad \frac{\quad}{\quad} + \quad \frac{\quad}{\quad} + \quad \frac{\quad}{\quad} + \frac{\quad}{\quad} \right]$$

    (b) Find a bound for the error made using this approximation.

    Hint: for the trapezoidal rule $E_n^T \leq \dfrac{K_T}{12n^2}(b-a)^3$, with $K_T = \max\limits_{a \leq x \leq b} |f''(x)|$

Values of the trigonometric integrals for some angles:

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	0	$\dfrac{1}{2}$	$\dfrac{1}{\sqrt{2}}$	$\dfrac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\dfrac{\sqrt{3}}{2}$	$\dfrac{1}{\sqrt{2}}$	$\dfrac{1}{2}$	0
$\tan\theta$	0	$\dfrac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined