1 a)

$$\int_0^{\pi/2} \sin^2 x \cos^2 x\,dx = \int_0^{\pi/2} \left(\frac{1-\cos(2x)}{2}\right)\left(\frac{1+\cos(2x)}{2}\right)dx$$

3pt

$$= \frac{1}{4}\int_0^{\pi/2} (1-\cos^2(2x))\,dx = \frac{1}{4}\int_0^{\pi/2} dx - \frac{1}{4}\int_0^{\pi/2} \cos^2(2x)\,dx$$

$$= \frac{1}{4}\int_0^{\pi/2} dx - \frac{1}{4}\int_0^{\pi/2} \frac{1+\cos(4x)}{2}\,dx = \frac{1}{4}\int_0^{\pi/2} dx - \frac{1}{8}\int_0^{\pi/2} dx - \frac{1}{8}\int_0^{\pi/2} \cos(4x)\,dx$$

4pt

$$= \frac{1}{8}\int_0^{\pi/2} dx - \frac{1}{8}\int_0^{\pi/2} \cos(4x)\,dx = \frac{1}{8}\left[x - \frac{1}{4}\sin(4x)\right]_0^{\pi/2}$$

3pt

$$= \frac{1}{8}\left[\left(\frac{\pi}{2} - \frac{1}{4}\cancel{\sin(2\pi)}^{\,0}\right) - \left(0 - \frac{1}{4}\cancel{\sin(0)}^{\,0}\right)\right] = \frac{\pi}{16}$$

3pt

b) $$\int \tan^5 x \sec^4 x\,dx = \int \tan^5 x \sec^2 x \sec^2 x\,dx = \int \tan^5 x(\tan^2 x + 1)\sec^2 x\,dx$$

$$u = \tan x$$
$$du = \sec^2 x\,dx$$

4pt

$$= \int u^5(u^2+1)\,du = \int (u^7 + u^5)\,du = \frac{u^8}{8} + \frac{u^6}{6} + C$$

3pt

$$= \frac{\tan^8 x}{8} + \frac{\tan^6 x}{6} + C$$

2 a.

$$\int \frac{x}{\sqrt{9-x^2}}\,dx = \int \frac{3\sin u}{\sqrt{9-9\sin^2 u}}\,3\cos u\,du = \int \frac{3\sin u}{3\sqrt{1-\sin^2 u}}\,3\cos u\,du$$

$x = 3\sin u$

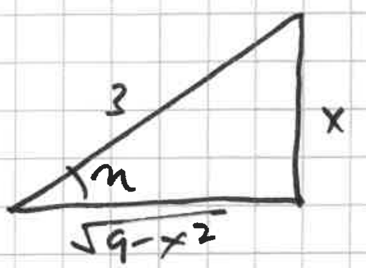
$dx = 3\cos u\,du$

3pt

$$= 3\int \frac{\sin u}{\sqrt{\cos^2 u}}\cos u\,du = 3\int \sin u\,du$$

4pt

$$= -3\cos u + C$$



Then $\displaystyle\int \frac{x}{\sqrt{9-x^2}}\,dx = -3\,\frac{\sqrt{9-x^2}}{3} + C$

$$= -\sqrt{9-x^2} + C$$

3pt

2 b.

$$\int_{\sqrt{2}}^{2} \frac{1}{x^2\sqrt{x^2-1}}\,dx = \int_{\pi/4}^{\pi/3} \frac{\sec u\tan u}{\sec^2 u\sqrt{\sec^2 u-1}}\,du$$

$x = \sec u$

$dx = \sec u\tan u\,du$

3pt

4pt

$$= \int_{\pi/4}^{\pi/3} \frac{\sec u\tan u}{\sec^2 u\sqrt{\tan^2 u}}\,du = \int_{\pi/4}^{\pi/3} \frac{1}{\sec u}\,du = \int_{\pi/4}^{\pi/3} \cos u\,du$$

3pt

$$= \left[\sin u\right]_{\pi/4}^{\pi/3} = \sin\left(\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}$$

3 a.

$$\int_0^1 \frac{x-1}{x^2+3x+2}\,dx$$

3pt

$$\frac{x-1}{x^2+3x+2} = \frac{x-1}{(x+2)(x+1)} = \frac{A}{x+2} + \frac{B}{x+1}$$

then

$x-1 = A(1+x) + B(x+2)$

for $x=-1$:

$\boxed{-2 = B}$

for $x=-2$:

$-3=-A$ then $\boxed{A=3}$ 2pt

then $\displaystyle\int_0^1 \frac{x-1}{x^2+3x+2}\,dx = \int_0^1 \left(\frac{3}{x+2} + \frac{-2}{x+1}\right) dx$

3pt

$$= \Big[3\ln(x+2) - 2\ln(x+1)\Big]_0^1 = \left[\ln\left(\frac{(x+2)^3}{(x+1)^2}\right)\right]_0^1$$

$$= \left[\ln\left(\frac{3^3}{2^2}\right) - \ln\left(\frac{2^3}{1}\right)\right] = \left[\ln\left(\frac{27}{4}\right) - \ln(8)\right]$$

2pt

$$= \ln\left[\frac{27}{32}\right]$$

3 b)

$$\int \frac{1}{x(x-1)^2}\,dx$$

$$\frac{1}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$ 3 pt

then $1 = A(x-1)^2 + Bx(x-1) + Cx$

for $x=1$: $\boxed{1 = C}$

for $x=0$: $\boxed{1 = A}$

then $1 = (x-1)^2 + Bx(x-1) + x$

for $x=2$, then $1 = 1 + 2B + 2 = 3 + 2B$

$-2 = 2B$ then $\boxed{B=-1}$ 2pt

2pt

$$\int \frac{1}{x(x-1)^2}\,dx = \int \frac{1}{x}\,dx + \int \frac{-1}{x-1}\,dx + \int \frac{1}{(x-1)^2}\,dx$$

Make $w = x-1$, $dw = dx$ Make $u = x-1$, $du = dx$

$$= \int \frac{dx}{x} - \int \frac{dw}{w} + \int \frac{du}{u^2} = \int \frac{dx}{x} - \int \frac{dw}{w} + \int u^{-2}\,du$$

3pt

$$= \ln|x| - \ln|w| - u^{-1} + C = \ln|x| - \ln|x-1| - (x-1)^{-1} + C$$

4.a)

Note

$$\int \frac{x}{(x^2+2)^2}\,dx = \frac{1}{2}\int \frac{du}{u^2} = \frac{1}{2}\int u^{-2}\,du = -\frac{1}{2}u^{-1}+c$$

5pt

Make $u = x^2+2$ $\qquad = -\frac{1}{2}(x^2+2)^{-1}+c$

$du = 2x\,dx$

2pt

$$\int_0^\infty \frac{x}{(x^2+2)^2}\,dx = \lim_{b\to\infty}\int_0^b \frac{x}{(x^2+2)^2}\,dx = \lim_{b\to\infty}\left[-\frac{1}{2}(x^2+2)^{-1}\right]_0^b$$

3pt

$$= \lim_{b\to\infty}\left[-\frac{1}{2(x^2+2)}\right]_0^b = \lim_{b\to\infty}\left[\frac{-1}{2(b^2+2)}+\frac{1}{2(2)}\right] = \left[0+\frac{1}{4}\right] = \frac{1}{4}.$$

b)

5pt

$$\int \frac{1}{x(\ln x)^{1/2}}\,dx = \int u^{-1/2}\,du = 2u^{1/2}+C = 2(\ln x)^{1/2}+C$$

$u = \ln x$

$du = \frac{dx}{x}$

2pt

$$\int_1^2 \frac{1}{x(\ln x)^{1/2}}\,dx = \lim_{a\to 1^+}\int_a^2 \frac{1}{x(\ln x)^{1/2}}\,dx = \lim_{a\to 1^+}\left[2(\ln x)^{1/2}\right]_a^2$$

3pt

$$= \lim_{a\to 1^+}\left[2(\ln 2)^{1/2} - 2(\ln a)^{1/2}\right] = 2(\ln 2)^{1/2}$$

5 a)

$a=2,\ b=4,\ n=4$

$$\int_2^4 \frac{1}{x}\,dx \approx \frac{b-a}{2n}\left[f(x_0)+2f(x_1)+2f(x_2)+2f(x_3)+f(x_4)\right]$$

$$=\frac{2}{2\cdot 4}\left[\frac{1}{2}+2\cdot\frac{1}{2.5}+2\cdot\frac{1}{3}+2\cdot\frac{1}{3.5}+\frac{1}{4}\right]$$

b) $f'(x)=-x^{-2}$

$f''(x)=2x^{-3}=\frac{2}{x^3}$

$$K_T=\max_{2\le x\le 4}\left|\frac{2}{x^3}\right| = \left|\frac{2}{2^3}\right| = \left|\frac{1}{2^2}\right| = \frac{1}{4}$$

then

$$E_4^T \le K_T\,\frac{1}{12n^2}(b-a)^3 = \frac{1}{4}\,\frac{1}{12\cdot 16}\,(2)^3 = \frac{1}{96}$$