MATH141 Fall 2019 Exam 4 [100 pt]

Instructions: Number the answer sheets from 1 to 5 and fill out all the information in each of them (sign the Honor Pledge on page 1 only). Solve only one problem in every answer sheet. If you need more space to solve a given problem, use the back of the same answer sheet. No lecture notes, cheat sheets, books, or electronic devices of any kind are allowed.

1. [20 pt] Evaluate the limit as a number, ∞ , or $-\infty$

(a)
$$\lim_{n \to \infty} \left(1 + \frac{1}{3n} \right)^n$$

(b)
$$\lim_{n \to \infty} \left(\frac{5n^2 + 2n + 1}{4 - 2n - 3n^2} \right)$$

2. [20 pt] Find the sum of the following series. For the second series, what is the j-th truncation error?

(a)
$$\sum_{n=2}^{\infty} \frac{2}{n^2 + 2n}$$

(b) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{2}{3}\right)^{2n}$

3. [20 pt] Use the comparison test, the limit comparison test or the integral test to determine if the series converges or diverges

(a)
$$\sum_{n=1}^{\infty} \frac{1}{n \sqrt[n]{n}}$$

(b) $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^2 + 1}$
(c) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2 + n - 1}}$

4. [20 pt] Determine which series diverge, which converge conditionally and which converge absolutely

(a)
$$\sum_{n=3}^{\infty} (-1)^{n+1} \frac{1}{(n+1)(n-2)}$$

(b) $\sum_{n=5}^{\infty} (-1)^{n+1} \left(\frac{n^3+3n+2}{3n^3-2n+1}\right)^n$
(c) $\sum_{n=1}^{\infty} (-1)^n n \left(\frac{2}{3}\right)^n$

5. [20 pt] Find the interval of convergence for the given series

(a)
$$\sum_{n=0}^{\infty} \frac{n!}{(3n)!} x^n$$

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{5^n (n+8)}$