Recall: $M_y = \int_a^b x\,[f(x) - g(x)]\,dx$, $M_x = \int_a^b \frac{1}{2}[(f(x))^2 - (g(x))^2]\,dx$

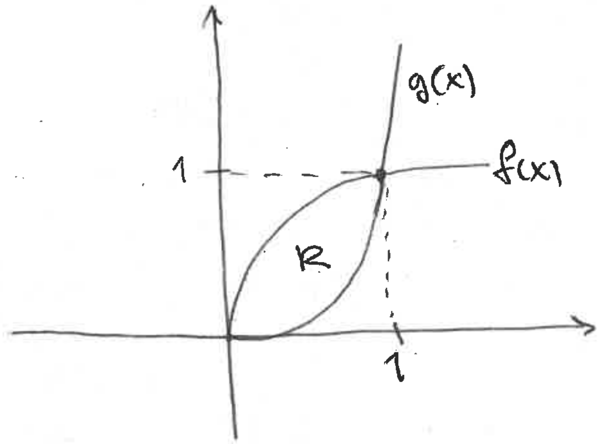
$A = \int_a^b [f(x) - g(x)]\,dx$ if $f(x) \geq g(x)$ on $[a, b]$.

MATH141 Summer I QUIZ 2

Name:

1. (6 pt) Calculate the center of gravity of the region R between the graphs of $f(x) = \sqrt{x}$ and $g(x) = x^2$. (Hint: the curves intersect at the points $(0, 0)$ and $(1, 1)$)

The region R is



$$M_y = \int_0^1 x(\sqrt{x} - x^2)\,dx = \int_0^1 x(x^{1/2} - x^2)\,dx = \int_0^1 (x^{3/2} - x^3)\,dx = \left[\frac{x^{5/2}}{5/2} - \frac{x^4}{4}\right]_0^1$$

$$= \frac{2}{5} - \frac{1}{4} = \frac{8-5}{20} = \frac{3}{20}$$

$$M_x = \int_0^1 \frac{1}{2}[(\sqrt{x})^2 - (x^2)^2]\,dx = \frac{1}{2}\int_0^1 [x - x^4]\,dx = \frac{1}{2}\left[\frac{x^2}{2} - \frac{x^5}{5}\right]_0^1 = \frac{1}{2}\left[\frac{1}{2} - \frac{1}{5}\right] = \frac{3}{20}$$

$$A = \int_0^1 (\sqrt{x} - x^2)\,dx = \int_0^1 (x^{1/2} - x^2)\,dx = \left[\frac{x^{3/2}}{3/2} - \frac{x^3}{3}\right]_0^1 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

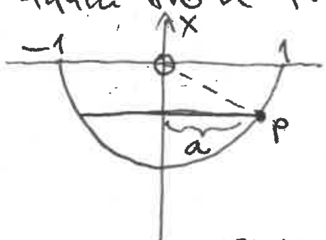
then $\bar{x} = \frac{M_y}{A} = \frac{9}{20}$ $\qquad \bar{y} = \frac{M_x}{A} = \frac{9}{20}$

the formula for ~~the~~ work required to pump the water between levels a and b to a heigh ℓ from a tank of cross-sectional area $A(x)$ is:

$$W = \int_a^b 62.5\, A(x)(\ell - x)\,dx.$$

2. (4 pt) Suppose a tank has the shape of a half cylinder 2 feet in diameter and 1 feet wide. If the tank is full of water, find the work W necessary to pump all the water to the top of the tank.

the tank from the side looks like:



Because the width of the tank is 1, the area of the cross-section of the tank is $A(x) = 2a\,(1) = 2a$

Each cross-section is a rectangle of sides $2a$ (see figure) and 1.

Because the point p lies on a circle, of radius 1 we have that $(-x)^2 + a^2 = (1)^2$. We are assuming that the top of the tank is at $x=0$.

then $A(x) = 2a = 2\sqrt{1-x^2}$.

the work required to pump out all the water is:

$$W = \int_{-1}^{0} 62.5\left[2\sqrt{1-x^2}\right](0-x)\,dx = 62.5\int_{-1}^{0}(-2x)\sqrt{1-x^2}\,dx$$

If we make $u = 1-x^2$ then $du = -2x\,dx$, then:

$$W = \int_0^1 62.5\, u^{1/2}\,du = 62.5\left[\frac{u^{3/2}}{3/2}\right]_0^1 = \frac{2}{3}(62.5) = \frac{125}{3} \text{ ft-lb}$$