MATH141 QUIZ 3

Name:

1. [5 pt] Consider the function $f(x) = x^x$ for $x > 0$.
   (a) Find the largest interval containing $x = 1$ on which f is invertible. (Hint: Find the largest interval on which the sign of f' does not change)
   (b) Calculate $(f^{-1})'(4)$

a) $f(x) = x^x = e^{x\ln x}$

$f'(x) = e^{x\ln x}\left[1 \cdot \ln x + \frac{x}{x}\right] = e^{x\ln x}\left[\ln x + 1\right]$

Note that $f'(1) = e^0\left[\ln(1) + 1\right] = 1 > 0$.

$f'(x) \geq 0$ if:

(*) $e^{x\ln x}\left[\ln x + 1\right] \geq 0$

Because $e^{x\ln x} > 0$ for any x, (*) happens if $\ln x + 1 \geq 0$,

that is, if $\ln x \geq -1$. Then we need that x is in $(\frac{1}{e}, \infty)$.

The largest interval containing $x=1$, in which f is invertible is then $(1/e, \infty)$.

b) Note that $f(2) = 2^2 = 4$, then $(f^{-1})'(4) = \frac{1}{f'(2)}$

$= \frac{1}{e^{2\ln(2)}\left[\ln(2) + 1\right]} = \frac{1}{4\left[\ln(2) + 1\right]}$

2. [5 pt] Solve the following integrals

(a) $\int 3^x dx = \int e^{x\ln 3} dx = \frac{1}{\ln 3}\int e^u du = \frac{1}{\ln 3} e^u + c$

$\boxed{\text{Make } u = x\ln 3}$

$= \frac{1}{\ln 3} e^{x\ln 3} + c = \frac{1}{\ln 3} 3^x + c$

(b) $\int 3^x \cdot 4^x dx$

$= \int e^{x\ln 3} e^{x\ln 4} dx = \int e^{x\ln 3 + x\ln 4} dx = \int e^{x(\ln 3 + \ln 4)} dx$

$\boxed{\text{Make } u = x(\ln 3 + \ln 4)}$

$= \frac{1}{\ln 3 + \ln 4} e^u + c = \frac{1}{\ln 3 + \ln 4} 3^x \cdot 4^x + c$

(c) $\int 2x \cdot 2^{x^2+5} dx$

$= \int 2x e^{(x^2+5)\ln 2} dx = \frac{1}{\ln 2}\int e^u du = \frac{1}{\ln 2} e^u + c$

$\boxed{\text{Make } u = (x^2+5)\ln 2}$

$= \frac{1}{\ln 2} e^{(x^2+5)\ln 2} + c = \frac{1}{\ln 2} 2^{x^2+5} + c$

(d) $\int \frac{1}{x} \log_5 x \, dx$

$= \int \frac{1}{x} \frac{\ln x}{\ln 5} dx = \frac{1}{\ln 5}\int \frac{1}{x} \ln x \, dx = \frac{1}{\ln 5}\int u \, du = \frac{1}{\ln 5}\frac{u^2}{2} + c = \frac{1}{\ln 5}\frac{(\ln x)^2}{2} + c$

$\boxed{\text{Make } u = \ln x}$

(e) $\int \frac{1}{x} \log_2 x \log_5 x \, dx$

$= \int \frac{1}{x} \frac{\ln x}{\ln 2} \frac{\ln x}{\ln 5} dx = \frac{1}{\ln 2 \ln 5}\int \frac{1}{x}(\ln x)^2 dx = \frac{1}{\ln 2 \ln 5}\int u^2 du$

$\boxed{\text{Make } u = \ln x}$

$= \frac{1}{\ln 2 \ln 5}\frac{u^3}{3} + c = \frac{1}{\ln 2 \ln 5}\frac{(\ln x)^3}{3} + c$.