MATH141 QUIZ 4

Name:

Calculate the following integrals or limits.

1. [2 pt] $\int \frac{\sec^2 x}{\sqrt{9-\tan^2 x}}dx = \int \frac{3\,du}{\sqrt{9-9u^2}} = \int \frac{du}{\sqrt{1-u^2}}$

$du = \frac{1}{3}\sec^2 x\,dx$

$= \sin^{-1} u + C = \sin^{-1}\left(\frac{1}{3}\tan x\right) + C$

2. [2 pt] $\int_0^1 \frac{\tan^{-1}(t)}{1+t^2}dt$ Make $u = \tan^{-1}(t)$
$du = \frac{1}{t^2+1}dt$

for $t=0$, then $u=0$, for $t=1$, then $u=\pi/4$, then

$\int_0^1 \frac{\tan^{-1}(t)}{1+t^2}dt = \int_0^{\pi/4} u\,du = \left.\frac{u^2}{2}\right|_0^{\pi/4} = \frac{1}{2}\left(\frac{\pi}{4}\right)^2 = \frac{1}{2}\frac{\pi^2}{16} = \frac{\pi^2}{32}.$

by l'Hospitals rule,
because ~~...~~ all the limits exist

3. [1 pt] $\lim_{x\to\infty} \frac{e^{2x}}{x^2+1} = \lim_{x\to\infty} \frac{2e^{2x}}{2x} = \lim_{x\to\infty} \frac{4e^{2x}}{2} = \infty$

4. [1 pt] $\lim_{x\to 0^+} x^{\sin x}$ $= \lim_{x\to 0^+} e^{\sin x \ln x}$

By l'Hospital's rule (because all limits exist)

$\lim_{x\to 0^+} \sin x \ln x = \lim_{x\to 0^+} \frac{\ln x}{\csc x} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\csc x \cot x}$

$= \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{\cos x}{\sin^2 x}} = \lim_{x\to 0^+} -\frac{\sin^2 x}{x\cos x} = \lim_{x\to 0^+} -\frac{\sin x}{x}\cdot\frac{x}{\cos x} = \lim_{x\to 0^+} -\frac{\sin x}{x} \cdot \lim_{x\to 0^+}\frac{x}{\cos x}$

$= -1\cdot 0 = 0$ then $\lim_{x\to 0^+} x^{\sin x} = e^{\lim_{x\to 0^+}\sin x \ln x} = e^0 = 1$

5. [2 pt] $\int x\cos x\,dx$ $= x\sin x - \int \sin x\,dx = x\sin x + \cos x + C$

$u = x$ $\quad dv = \cos x\,dx$
$du = dx$ $\quad v = \sin x$

6. [2 pt] $\int_1^e x\ln x^2 dx$ $= \frac{x^2}{2}\ln x^2\Big|_1^e - \int_1^e \frac{x^2}{2}\frac{2}{x}dx$

$$\boxed{\begin{aligned} &u = \ln x^2 \quad dv = x\,dx \\ &du = \frac{1}{x^2}2x\,dx \\ &\quad = \frac{2}{x}dx \qquad v = \frac{x^2}{2}\end{aligned}}$$

$= \frac{x^2}{2}\ln x^2\Big|_1^e - \int_1^e x\,dx$

$= \frac{x^2}{2}\ln x^2\Big|_1^e - \frac{x^2}{2}\Big|_1^e$

$= \frac{e^2}{2}\ln(e^2) - \cancel{\frac{(1)^2}{2}\ln(1^2)}^{\to 0} - \left(\frac{e^2}{2} - \frac{1^2}{2}\right)$

$= \frac{e^2}{2}2\cdot\ln(e) - \frac{e^2}{2} + \frac{1}{2} = e^2 - \frac{e^2}{2} + \frac{1}{2} = \frac{e^2}{2} + \frac{1}{2}.$