MATH141 QUIZ 5

Name:

1. Evaluate the integral $\int \tan^5 x \sec^2 x dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$$\int \tan^5 \sec^2 x \, dx = \int u^5 \, du = \frac{u^6}{6} + C = \frac{\tan^6 x}{6} + C$$

2. Evaluate the integral $\int \frac{x^2}{\sqrt{1-x^2}} dx$

$x = \sin u$

$dx = \cos u \, du$

$$\int \frac{x^2}{\sqrt{1-x^2}} dx = \int \frac{\sin^2 u}{\sqrt{1-\sin^2 u}} \cos u \, du = \int \frac{\sin^2 u}{\sqrt{\cos^2 u}} \cos u \, du$$

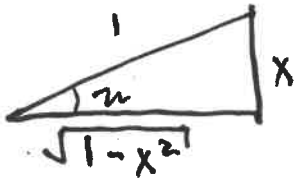
$$= \int \sin^2 u \, du = \int \frac{1-\cos(2u)}{2} du = \frac{1}{2}\int du - \frac{1}{2}\int \cos(2u) du$$

$w = 2u$

$dw = 2du$

$$= \frac{1}{2}u - \frac{1}{4}\int \cos(w) dw = \frac{1}{2}u - \frac{1}{4}\sin(w) + C = \frac{1}{2}u - \frac{1}{4}\sin(2u) + C$$

$$= \frac{1}{2}u - \frac{1}{2}\sin u \cos u + C = \frac{1}{2}\sin^{-1} x - \frac{1}{2} x \sqrt{1-x^2} + C$$



3. Evaluate the integral $\int \frac{2x+5}{x^2+3x+2}dx$

$$\frac{2x+5}{x^2+3x+2} = \frac{2x+5}{(x+2)(x+1)} = \frac{A}{x+2} + \frac{B}{x+1}$$

then (after multiplying by $(x+2)(x+1)$):

$$2x+5 = A(x+1) + B(x+2)$$

for $x=-1$: $\quad 2(-1)+5 = A(-1+1) + B(-1+2)$

$$3 = 0 + B$$

then $\boxed{B=3}$

for $x=-2$: $\quad 2(-2)+5 = A(-2+1) + B(-2+2)$

$$1 = -A + 0$$

then $\boxed{A=-1}$

then

$$\int \frac{2x+5}{x^2+3x+2}dx = \int \frac{-1}{x+2}dx + \int \frac{3}{x+1}dx$$

$$\therefore = -\ln|x+2| + 3\ln|x+1| + C$$