MATH141 QUIZ 6

Name:

1. Determine whether the improper integral converges. If it does, determine the value of the integral

(a) $\int_0^1 \frac{e^t}{\sqrt{e^t-1}}$

Note $\int \frac{e^t}{\sqrt{e^t-1}}dt = \int \frac{du}{u^{1/2}} = \int u^{-1/2}du = 2u^{1/2}+c$

Make $u=e^t-1, du=e^t dt$ $= 2(e^t-1)^{1/2}+c$

then:

$\lim_{c^+\to 0}\int_c^1 \frac{e^t}{\sqrt{e^t-1}}dt = \lim_{c^+\to 0}\left[2(e^t-1)^{1/2}\right]_c^1 = \lim_{c^+\to 0}\left[2(e-1)^{1/2}-2(e^c-1)^{1/2}\right]$

$= \left[2(e-1)^{1/2}-0\right] = 2(e-1)^{1/2}$ the integral converges

(b) $\int_{-\infty}^{\infty} \frac{x^3}{(x^4+1)^2}$

Note $\int \frac{x^3}{(x^4+1)^2} = \frac{1}{4}\int\frac{du}{u^2} = \frac{1}{4}\int u^{-2}du = -\frac{1}{4}u^{-1}+c = -\frac{1}{4}(x^4+1)^{-1}+c$

make $u=x^4+1$
$du=4x^3dx$

then:

$\lim_{b\to\infty}\int_0^\infty \frac{x^3}{(x^4+1)^2}dx = \lim_{b\to\infty}\left[-\frac{1}{4}(x^4+1)^{-1}\right]_0^b = \lim_{b\to\infty}\left[-\frac{1}{4}(b^4+1)^{-1}-\left(-\frac{1}{4}(0^4+1)\right)\right] = +\frac{1}{4}$

$\lim_{a\to-\infty}\int_a^0 \frac{x^3}{(x^4+1)^2}dx = \lim_{a\to-\infty}\left[-\frac{1}{4}(x^4+1)^{-1}\right]_a^0 = \lim_{a\to-\infty}\left[-\frac{1}{4}(0^4+1)-\left(-\frac{1}{4}(a^4+1)\right)\right] = -\frac{1}{4}$

then $\int_{-\infty}^{\infty}\frac{x^3}{(x^4+1)^2} = \lim_{a\to-\infty}\int_a^0\frac{x^3}{(x^4+1)^2}dx + \lim_{b\to\infty}\int_0^b\frac{x^3}{(x^4+1)}dx =$

$= -\frac{1}{4}+\frac{1}{4} = 0.$ the integral converges.

2. Find a formula for the n-th Taylor polynomial of f:

$$f(x) = \frac{1}{1+x} = (1+x)^{-1}$$

$f'(x) = (-1)(1+x)^{-2}$ $\qquad f'(0) = -1$

$f''(x) = (-1)(-2)(1+x)^{-3}$ $\qquad f''(0) = 2$

$f'''(x) = (-1)(-2)(-3)(1+x)^{-4}$ $\qquad f'''(0) = (-1)^3 3! = -3!$

$\vdots$

$f^{(n)}(x) = (-1)(-2)(-3)\ldots(-n)(1+x)^{-n-1}$ $\qquad f^{(n)}(0) = (-1)^n n!$

$= (-1)^n n! (1+x)^{-n-1}$

then $p_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \ldots + \frac{f^{(n)}(0)}{n!}x^n$

~~and~~

$= 1 - x + \frac{2}{2!}x^2 + \ldots + \frac{(-1)^n n!}{n!}x^n$

$= 1 - x + x^2 + \ldots + (-1)^n x^n.$