MATH141 QUIZ 7

Name:

1. Find a formula for the j-th partial sum of the series. Find the sum of the series by calculating $\lim_{j\to\infty} s_j$.

   (a) $\sum_{n=1}^{\infty} \frac{2}{2n^2+2n}$

$$= \sum_{n=1}^{\infty} \frac{2}{2(n^2+n)} = \sum_{n=1}^{\infty} \frac{1}{n^2+n} = \sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$s_1 = 1 - \frac{1}{2}$$

$$s_2 = \left(1 - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \frac{1}{3}\right) = 1 - \frac{1}{3}$$

$$s_3 = \left(1 - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \frac{1}{4}\right) = 1 - \frac{1}{4}$$

$$s_j = \left(1 - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \dots + \left(\cancel{\frac{1}{j-1}} - \cancel{\frac{1}{j}}\right) + \left(\cancel{\frac{1}{j}} - \frac{1}{j+1}\right)$$

$$= 1 - \frac{1}{j+1}$$

$$\lim_{j\to\infty} s_j = \lim_{j\to\infty}\left(1 - \frac{1}{j+1}\right) = 1$$

then $$\sum_{n=1}^{\infty} \frac{2}{2n^2+2n} = 1$$

2. Determine whether or not the series converges. If it does, find its sum:

(a) $\sum_{n=2}^{\infty} \left(\frac{1}{\pi}\right)^{n+2}$

(b) $\sum_{n=3}^{\infty} (-1)^n \left[\frac{2^n - 3^{n-3}}{4^n}\right]$

a) converges because it is geometric with $\boxed{r = \frac{1}{\pi}}$.

Then $\sum_{n=2}^{\infty} \left(\frac{1}{\pi}\right)^{n+2} = \sum_{n=2}^{\infty} \left(\frac{1}{\pi}\right)^2 \left(\frac{1}{\pi}\right)^n = \frac{cr^m}{1-r} = \frac{\left(\frac{1}{\pi}\right)^2\left(\frac{1}{\pi}\right)^2}{1-\frac{1}{\pi}} = \frac{\left(\frac{1}{\pi}\right)^4}{1-\frac{1}{\pi}} = \frac{1}{\pi^4 - \pi^3}$

b) $\sum_{n=3}^{\infty} (-1)^n \left[\frac{2^n - 3^{n-3}}{4^n}\right] = \sum_{n=3}^{\infty} \left[\underbrace{(-1)^n \frac{2^n}{4^n}} + \underbrace{(-1)^n \left(-\frac{3^{n-3}}{4^n}\right)}\right]$ converges because these two converge.

$= \sum_{n=3}^{\infty} \left[\left(-\frac{1}{2}\right)^n + (-1)^{n+1} \frac{1}{3^3} \frac{3^n}{4^n}\right] = \sum_{n=3}^{\infty} \left[\left(-\frac{1}{2}\right)^n - \frac{1}{3^3}\left(-\frac{3}{4}\right)^n\right]$

$= \frac{\left(-\frac{1}{2}\right)^3}{1-\left(-\frac{1}{2}\right)} - \frac{\left(\frac{1}{3^3}\right)\left(-\frac{3}{4}\right)^3}{1-\left(-\frac{3}{4}\right)} = \frac{-\frac{1}{8}}{\frac{3}{2}} + \frac{\left(\frac{1}{3^3}\right)\left(\frac{3}{4}\right)^3}{\frac{7}{4}}$

$= -\frac{1}{12} + \frac{\frac{1}{4^3}}{\frac{7}{4}} = -\frac{1}{12} + \frac{1}{4^2 \cdot 7}$