MATH246 Summer II
Exam 1 [100 pt]

Instructions: Number the answer sheets from 1 to 4 and fill out all the information in each of them (sign the Honor Pledge on page 1 only). Solve only one problem in every answer sheet. If you need more space to solve a given problem, use the back of the same answer sheet. No lecture notes, cheat sheets, books, or electronic devices of any kind are allowed.
For full credit, you need to evaluate any integral you encounter.

1. For each of the initial value problems find the largest interval where there is a unique solution and find the solution on that interval.

   (a) [10 pt]
   $$ty' + 2y = 4t^2 - 3t \qquad y(1) = 0$$

   (b) [15 pt]
   $$(t^2 - 1)y' + 4ty = \frac{1}{(t^2 - 1)} \qquad y(0) = 1$$

   Hint: You should find that an integrating factor is $(t^2 - 1)^2$.

2. Find the solution of the initial value problems in explicit form and say what the largest interval of definition for each of these solutions is.

   (a) [10 pt]
   $$y' = \frac{16e^{2x} - 4e^x}{2y + 2} \qquad y(0) = 1$$

   (b) [15 pt]
   $$y' = \frac{1}{2}\frac{1}{(y + xy)} \qquad y(0) = -1$$

3. Consider the following differential equation:
   $$(y + 6x^2)dx + (x \ln x - 2x)dy = 0 \qquad x > 0$$

   (a) [5 pt] Show that the equation is not exact
   (b) [10 pt] Show that $\mu(x) = \frac{1}{x}$ is an integrating factor for the equation by writing an equation for μ and solving it. You can assume already that μ depends on x alone.
   (c) [10 pt] Find the general solution of the differential equation.

4. [25 pt] In the absence of predators the population of mosquitoes in a certain area would increase at a rate proportional to its population according to the following formula
   $$\frac{dp}{dt} = rp$$
   where p is the population of mosquitoes, $r = 0.1$ and the time t is measured in weeks. (This value of r implies that the population of mosquitoes doubles approximately every three weeks.) There are 100,000 mosquitoes in the area initially when a flock of birds arrives that eats 9,000 mosquitoes per week.

   (a) Write an initial value problem that models this situation and find a formula for the number of mosquitoes in terms of t.
   (b) How many mosquitoes would the flock need to eat weekly, from the time it arrives, for the population of mosquitoes to be decreasing in time?