DEPARTMENT OF MATHEMATICS

Student's Name________ Course#______ Prob.# 1 Date______

Section Instructor________ Sec.#______

GRADING

HONOR PLEDGE: I pledge on my honor that I have not given or received any unauthorized assistance on this examination or assignment.

Please write the exact wording of the pledge, followed by your signature, in the space below:

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a) Rewrite:

$$y' + \frac{2}{t} y = 4t - 3 \qquad y(1) = 0$$

the only value that makes the coefficient indetermined is $x=0$. Then the largest interval of definition that includes the initial condition is $(0,\infty)$. (1 pt)

An integrating factor is $e^{\int \frac{2}{t} dt} = e^{2\ln t} = e^{\ln t^2} = t^2$

then

$$\frac{d}{dt}(t^2 y) = (4t-3)t^2 = 4t^3 - 3t^2$$

$$\boxed{t^2 y = t^4 - t^3 + c} \qquad (5\ pt)$$

Because $y(1)=0$;

$$0 = (1)^4 - (1)^3 + c, \text{ then } c = 0.$$

the solution is

$$\boxed{y = t^2 - t} \qquad (4\ pt)$$

b)

Rewrite:

$$y' + \frac{4t}{(t^2-1)}\, y = \frac{1}{(t^2-1)^2} \qquad y(0)=1$$

The values that make the coefficient or forcing undetermined are $t=1$ or $t=-1$. Then the largest interval of definition for the solution given the initial condition is $(-1,1)$

(1 pt)

An integrating factor is

$$e^{\int \frac{4t}{(t^2-1)}dt} = e^{2\int \frac{2t}{(t^2-1)}dt} = e^{2\ln(t^2-1)} = (t^2-1)^2$$

then

$$\frac{d}{dt}\left((t^2-1)^2 y\right) = \frac{1}{(t^2-1)^2}(t^2-1)^2 = 1$$

then

$$\boxed{(t^2-1)^2 y = t + c}$$ (8 pt)

Because $y(0)=1$; $(0^2-1)^2(1) = (0) + c$ then $c=1$.

the solution is

(6 pt)

$$\boxed{y = \frac{t+1}{(t^2-1)^2}}$$

DEPARTMENT OF MATHEMATICS

Student's Name__________ Course#______ Prob.# 2 Date______
Section Instructor__________ Sec.#______

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a) Separate the equation:

$$\int (2y+2)\,dy = \int (16e^{2x} - 4e^{x})\,dx$$

then $\boxed{y^2 + 2y = 8e^{2x} - 4e^{x} + C}$ —— (4 pt)

Because $y(0)=1$: $(1)^2 + 2(1) = 8e^0 - 4e^0 + C = 4 + C$

then $C = -1$

Then $\boxed{y^2 + 2y = 8e^{2x} - 4e^{x} - 1}$ (4 pt)

$$y^2 + 2y + 1 = 8e^{2x} - 4e^{x}$$

$$(y+1)^2 = 8e^{2x} - 4e^{x}$$

then $\boxed{y = -1 + \sqrt{8e^{2x} - 4e^{x}}}$ where the sign was chosed based on $y(0) = 1$.
(1 pt)

$$y = -1 + \sqrt{4(2e^{2x} - e^{x})} = -1 + 2\sqrt{(2e^{2x} - e^{x})}$$

we need that $2e^{2x} - e^{x} > 0$
that is $e^{x}(2e^{x} - 1) > 0$.
$e^{x} > 0$ always so we only need $2e^{x} - 1 > 0$

then $2e^{x} > 1$ (1 pt)
$e^{x} > \frac{1}{2}$
then $x > \ln(\frac{1}{2})$.
The largest interval of definition is $(\ln(\frac{1}{2}), \infty)$

b)

Separate

$$\int 2y\,dy = \int \frac{dx}{1+x}$$

then $\boxed{y^2 = \ln|1+x| + c}$ (6 pt)

Because of $y(0) = -1$: $(-1)^2 = \ln|1+0| + c = c$

$\boxed{\text{then } c = 1}$ (6 pt)

then $y = \pm\sqrt{\ln|1+x| + 1}$

Because of the initial condition we need to take the negative sign: $\boxed{y = -\sqrt{\ln|1+x|+1}}$

(2 pt)

$x = -1$ cannot be in the domain of definition of this formula, but we know that $x=0$ is (because of the initial condition).

Then the largest interval of definition are all the values of x in $(-1, \infty)$ such that

$$\ln(1+x) + 1 > 0$$

i.e.

$$\ln(1+x) > -1$$

$$1+x > e^{-1}$$

$$x > e^{-1} - 1$$

Then the largest interval of definition for the formula is (1 pt)

$$(e^{-1} - 1, \infty)$$

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Student's Name__________ Course#______ Prob.# 3 Date______

Section Instructor__________ Sec.#______

GRADING

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a)

$$\partial_y M = \partial_y (y + 6x^2) = 1$$

$$\partial_x N = \partial_x (x \ln x - 2x) = \ln x + x \frac{1}{x} - 2 = \ln x - 1$$

then the equation is not exact (5 pt)

b) We need

$$\partial_y (\mu (y + 6x^2)) = \partial_x (\mu (x \ln x - 2x))$$

As $\mu = \mu(x)$, we have

$$\mu \partial_y (y + 6x^2) = \mu' (x \ln x - 2x) + \mu (\ln x - 1)$$

$$\mu = \mu' (x \ln x - 2x) + \mu (\ln x - 1)$$

$$0 = \mu' (x \ln x - 2x) + \mu (\ln x - 2)$$

$$0 = \mu' (\ln x - 2) x + \mu (\ln x - 2)$$

$$0 = \mu' x + \mu \quad \text{———} \quad (6 \text{ pt})$$

then $\frac{d\mu}{dx} = -\frac{\mu}{x}$ then $\frac{d\mu}{\mu} = -\frac{dx}{x}$

then $\ln|\mu| = -\ln|x|$ then $|\mu| = \frac{1}{|x|}$ (4 pt)

So that $\mu = \frac{1}{x}$ is an integrating factor

c) The equation to solve is

$$\frac{1}{x}(y + 6x^2)\,dx + \frac{1}{x}(x\ln x - 2x)\,dy = 0$$

$$\left(\frac{y}{x} + 6x\right)dx + (\ln x - 2)\,dy = 0$$

$$H(x,y) = \int \left(\frac{y}{x} + 6x\right)dx = y\ln x + 3x^2 + h(y) \qquad (5\text{ pt})$$

We need

$$\partial_y H = \ln x - 2$$

$$\partial_y \left(y \ln x + 3x^2 + h(y)\right) = \ln x - 2$$

$$\ln x + h'(y) = \ln x - 2$$

then $h'(y) = -2$, then $h(y) = -2y$.

The general solution is

$$y\ln x + 3x^2 - 2y = C \qquad (5\text{ pt})$$

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Student's Name__________ Course#______ Prob.# 4 Date______

Section Instructor__________ Sec.#______

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a)

The initial value problem to solve is

$$\frac{dp}{dt} = 0.1\, p - 9000 \qquad p(0) = 100\,000$$

(5 pt)

Rewrite:

$$\frac{dp}{dt} - 0.1\, p = -9000$$

An integrating factor is $e^{\int(-0.1)\,dt} = e^{-0.1t}$

then $$\frac{d}{dt}\left(e^{-0.1t}\, p\right) = -9000\, e^{-0.1t}$$

then $$e^{-0.1t}\, p = \frac{-9000}{-0.1}\, e^{-0.1t} + C$$

then $$\boxed{e^{-0.1t}\, p = 90000\, e^{-0.1t} + C}$$ ——— (10 pt)

Because $p(0) = 100\,000$:

$$(e^0)\,100000 = 90\,000(e^0) + C \quad \text{then} \quad C = 10\,000$$

The required formula is (solving for p):

$$p = \left[90000\,e^{-0.1t} + 10\,000\right] e^{0.1t}$$

$$\boxed{p = 90000 + 10\,000\,e^{0.1t}} \qquad (5pt)$$

b) (5 pt) If the rate at which the flock can eat mosquitoes is represented by A, then (from a)):

$$e^{-0.1t}\,p = \frac{A}{0.1}\,e^{-0.1t} + C$$

so that

$$p = \frac{A}{0.1} + Ce^{0.1t}$$

For the population to be decreasing we need that $C < 0$.
At time $t=0$ we have:

$$100\,000 = p(0) = \frac{A}{0.1} + C \quad \text{then} \quad C = 100000 - \frac{A}{0.1}$$

then we need that

$$100000 - \frac{A}{0.1} < 0$$

then $\quad 10\,000 < A$

The birds need to eat more than 10000 mosquitoes weekly.