MATH246 Summer II
Exam 2 [100 pt]

Instructions: Number the answer sheets from 1 to 4 and fill out all the information in each of them (sign the Honor Pledge on page 1 only). Solve only one problem in every answer sheet. If you need more space to solve a given problem, use the back of the same answer sheet. No lecture notes, cheat sheets, books, or electronic devices of any kind are allowed.
For full credit, you need to evaluate any integral you encounter.

1. (a) [9pts] Let L be a linear ordinary differential operator with constant coefficients. Suppose that its characteristic polynomial can be factored as

$$p(z) = z(z+3)^2(z^2-4z+13).$$

Give a general real solution of the homogeneous equation $Ly = 0$.

(b) [6pts] The functions $Y_1(t) = 1 - t$ and $Y_2(t) = e^{-t}$ solve

$$ty'' - (1-t)y' - y = 0, \qquad t > 0$$

(you do not need to check this fact). Compute the Wronskian $W[Y_1, Y_2](t)$ and give the general solution of this equation.

(c) [10pts] State the largest interval on which a solution to the initial-value problem is determined by the conditions given

$$(x^2-1)y'' + \frac{y}{x-3} = e^x \cos(x), \qquad y(2) = y'(2) = \pi$$

2. [25 pt] Find the general solution of

$$y'' - 16y = 32e^{-4t}$$

3. [25pts] Find the general solution of

$$y'' - 5y' + 6y = 20\sin(4t) + 36t^2$$

Hint: show by any of the two methods that a particular solution to the problem $y'' - 5y' + 6y = 36t^2$ is $y_p = 6t^2 + 10t + \frac{38}{6}$.

4. The two parts of this problem are independent of each other.

(a) [12pts] The vertical displacement of an unforced mass on a spring is given by

$$h(t) = e^{-5t}\cos(6t) + e^{-5t}\sin(6t)$$

i. What value does h approach as t increases to infinity?
ii. Is this system undamped, under-damped, critically damped, or over-damped?
iii. Express $h(t)$ in the amplitude-phase form $h(t) = Ae^{-5t}\cos(6t - \delta)$ with $A > 0$ and $0 \le \delta \le 2\pi$ (the phase may be expressed in terms of an inverse trig function). Hint: recall that $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

(b) [13pts] The vertical displacement of a mass on a spring satisfies

$$h'' + 16h = 0$$

Find $h(t)$ for all $t > 0$ if the mass is set in motion at time $t = 0$ from its resting position with downward velocity -4.