MATH246 Summer II
QUIZ 1

Name:

(5pt)

1. Find the largest interval where the initial value problem has a unique solution and find the solution on that interval.

$$y' = 4e^{2x}; \qquad y(0) = 1$$

Note that this equation can be seen as a linear equation with coefficient equal to zero. The forcing $(4e^{2x})$ is never undefined, so the interval of definition of the solution is $(-\infty, \infty)$.

⊛ For the solution:

$$y = \int 4e^{2x}\,dx = 2e^{2x} + C$$

Because $y(0)=1$, then $1 = 2e^{0} + C$ and therefore $C=-1$.

The solution is $y = 2e^{2x} - 1$.

⊛ Alternatively, by the fundamental theorem of calculus

$$y = y(0) + \int_0^x 4e^{2s}\,ds = 1 + \int_0^x 4e^{2s}\,ds = 1 + \left[2e^{2s}\right]_0^x = 1 + 2e^{2x} - 2$$

$$= 2e^{2x} - 1$$

(5pt)

2. Find the largest interval where the initial value problem has a unique solution and find the solution on that interval.

$$x\frac{dy}{dx} + y = 3x + 2; \qquad y(1) = 3$$

Rewrite the equation as

$$(*) \quad \frac{dy}{dx} + \frac{y}{x} = \frac{3x+2}{x}$$

We see that $x=0$ cannot be in the interval of definition of y. the largest interval containing $x=1$ that does not include $x=0$ is $(0,\infty)$. Then the interval of definition of the solution to the initial-value problem is $(0,\infty)$.

An ~~the~~ integrating factor is $e^{\int \frac{1}{x}dx} = e^{\log x} = x$.

Multiply $(*)$ by x:

$$x\frac{dy}{dx} + y = \frac{3x^2+2x}{x} = 3x+2$$

$$\frac{d}{dx}(xy) = 3x+2$$

then $$xy = 1\cdot 3 + \int_1^x (3s+2)\,ds = 3 + \left[\frac{3}{2}s^2 + 2s\right]_1^x$$

$$= 3 + \left[\frac{3}{2}x^2 + 2x - \frac{3}{2} - 2\right] = \frac{3}{2}x^2 + 2x - \frac{1}{2}$$

then $$y = \frac{3}{2}x + 2 - \frac{1}{2x}.$$

Alternatively, find the resulting costant of integration c using that $y(1)=3$.