MATH246 Summer II
QUIZ 2

Name:

1. (5 pt) Find the solution of the initial value problem

$$\frac{dy}{dx} = \frac{2x+2}{2y-2} \qquad y(\tfrac{1}{4}) = \tfrac{1}{4}$$

and determine the interval in which the solution exists. Find an explicit formula for y.

The equation is separable: $\int (2y-2)\,dy = \int (2x+2)\,dx$

then $\boxed{y^2 - 2y = x^2 + 2x + c}$

Because of $y(\frac{1}{4}) = \frac{1}{4}$ we have: $\left(\frac{1}{4}\right)^2 - 2\left(\frac{1}{4}\right) = \left(\frac{1}{4}\right)^2 + 2\left(\frac{1}{4}\right) + c$,

then the solution is given implicitly by

$\boxed{y^2 - 2y = x^2 + 2x - 1}$

then $y^2 - 2y + 1 = x^2 + 2x$

$(y-1)^2 = x^2 + 2x$

Then $y = 1 \pm \sqrt{x^2+2x}$. Because $\sqrt{x^2+2x} \geq 0$, we need to choose the negative sign to satisfy $y(\frac{1}{4}) = \frac{1}{4}$.

$\boxed{y = 1 - \sqrt{x^2+2x}}$ the domain of definition of this formula are all the values of x such that $x^2+2x \geq 0$.

$x^2+2x \geq 0$ if $x \in (-\infty, -2]$ or $x \in [0, \infty)$. Only the second interval contains $x = \frac{1}{4}$, and because of the equation we need to exclude $x = 0$ (otherwise $y = 1$, and the right hand side is not defined). Then the interval in which the solution exists is $(0, \infty)$

2. (5 pt) Is the following differential form exact? If yes, find an implicit general solution to the differential equation. If not, find an integrating factor and then find an implicit general solution to the resulting differential equation.

$$\underbrace{(3xy+y^2)}_{M}dx+\underbrace{(x^2+xy)}_{N}dy=0$$

$\partial_y M = 3x+2y$

$\partial_x N = 2x+y$ $\boxed{\text{the differential form is not exact}}$

If μ is an integrating factor, then we need that:

$\partial_y\left[\mu(3xy+y^2)\right]=\partial_x\left[\mu(x^2+xy)\right]$

$\partial_y\mu(3xy+y^2)+\mu(3x+2y)=\partial_x\mu(x^2+xy)+\mu(2x+y)$

then

$\partial_y\mu(3x+y)y+\mu(x+y)=\partial_x\mu(x+y)x$

Because of the common factor $(x+y)$, it makes sense to try to find μ such that $\partial_y\mu=0$. That is, we want to find μ that depends on x alone. We need to solve: $\mu(x+y)=\mu' x(x+y)$

or $\mu=\mu' x$.

this is a separable equation: $\frac{dx}{x}=\frac{d\mu}{\mu}$, and we have $\ln|x|=\ln|\mu|$.

$\boxed{\text{then an integrating factor is } \mu=x}$.

the new differential form reads: (now it is exact):

$$\underbrace{(3x^2y+xy^2)}_{M}dx+\underbrace{(x^3+x^2y)}_{N}dy=0$$

$H(x,y)=\int M\,dx=\int(3x^2y+xy^2)dx=x^3y+\frac{1}{2}x^2y^2+h(y)$

We need:

$\partial_y H=N$

$x^3+x^2y+h'(y)=x^3+x^2y$.

We can take $h(y)=0$ (from the above equation, we know it is a constant) and then the implicit general solution is

$$\boxed{x^3y+\frac{1}{2}x^2y^2=C}$$