MATH246 Summer II
QUIZ 2

Name:

1. [4 pt] State the largest interval on which a solution to the given initial value problem is determined by the conditions given

$$(x^2-1)y''+y=e^x\cos(x), y(2)=y'(2)=\pi$$

Write as

$$y''+\frac{1}{(x^2-1)}y=\frac{e^x\cos(x)}{(x^2-1)}.$$

Coefficients and forcing are not defined if $x=1$ or $x=-1$
the largest interval that includes $x=2$ is then $(1,\infty)$.

2. [6 pt]

(a) Check that e^{3z} and e^{4z} are solutions to $w'' - 7w' + 12w = 0$.

(b) Check that $\{e^{3z}, e^{4z}\}$ is a fundamental set for this differential equation. Hint: We only need to show that its Wronskian satisfies $W[e^{3z}, e^{4z}] \neq 0$, because we already show that they are solutions to the equation.

(c) Use the method of superposition to find the solution $W(z)$ of the differential equation that satisfies $W(0) = 0, W'(0) = 0$. Hint: assume that $W(z) = c_1 e^{3z} + c_2 e^{4z}$ and then use the initial data to find c_1 and c_2.

a)
$$(e^{3z})'' - 7(e^{3z})' + 12e^{3z} = 9e^{3z} - 21e^{3z} + 12e^{3z} = 0$$
$$(e^{4z})'' - 7(e^{4z})' + 12e^{4z} = 16e^{4z} - 28e^{4z} + 12e^{4z} = 0$$

b)
$$W[e^{3z}, e^{4z}] = \det \begin{pmatrix} e^{3z} & e^{4z} \\ 3e^{3z} & 4e^{4z} \end{pmatrix} = 4e^{7z} - 3e^{7z} = e^{7z} \neq 0$$

c)
$$W'(z) = 3c_1 e^{3z} + 4c_2 e^{4z}$$

We need
$$0 = c_1 e^{3(0)} + c_2 e^{4(0)} = c_1 + c_2 \qquad \text{I}$$
$$0 = 3c_1 e^{3(0)} + 4c_2 e^{4(0)} = 3c_1 + 4c_2 \qquad \text{II}$$

Substracting I from II three times we have:

$$0 + c_2 = 0 \quad \text{, then } c_2 = 0$$

and using any of the two equations we obtain that $c_1 = 0$ too.

Alternatively, we note that $c_1 = 0, c_2 = 0$ is a solution to the system. ~~By~~ Because $W[e^{3z}, e^{4z}] \neq 0$, then that is the unique solution.