MATH246 Summer II
QUIZ 4

Name:

Find the general solution of any of the following differential equations (choose only one)

1.

$$y'' + y - 6 = \cos(2t)$$

We can rewrite this eq. as $y'' + y = 6 + \cos(2t)$

$\boxed{y_H}$ $y'' + y = 0$

the char. poly. is $p(z) = z^2 + 1$ with roots $\pm i$.

Then $y_H(t) = c_1 \cos(t) + c_2 \sin(t)$

$\boxed{y_p}$ To find a particular solution, separate into two parts:

$\boxed{y_p \mid \mathrm{I}}$ $y'' + y = 6$

then $y_{p_I}(t) = 6$

$\boxed{y_p \mid \mathrm{II}}$ $y'' + y = \cos(2t)$

For the forcing:

$\boxed{\begin{array}{c} d = 0 \\ \mu + i\nu = 2i \\ m = 0 \end{array}}$

$L(e^{2it}) = p(2i)e^{2it} = [(2i)^2 + 1]e^{2it} = -3e^{2it}$

then $L\left(\frac{e^{2it}}{-3}\right) = e^{2it}$

then $y_{p_{II}}(t) = \mathrm{Re}\left(\frac{e^{2it}}{-3}\right) = -\frac{1}{3}\cos(2t)$.

$\boxed{\text{General Solution}}$

$y(t) = y_H(t) + y_{p_I}(t) + y_{p_{II}}(t) = c_1 \cos(t) + c_2 \sin(t) + 6 - \frac{1}{3}\cos(2t)$

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Name:

Find the general solution of any of the following differential equations (choose only one)

1.

$$y'' + y - 6 = \cos(2t)$$

Alternatively find the particular solutions by undetermined coefficients:

$\boxed{y_p \mid I}$ $y'' + y = 6$

then $y_{p_I}(t) = 6$

$\boxed{y_p \mid II}$ $y'' + y = \cos(2t)$

$\boxed{\begin{matrix} \nu = 0 \\ \mu + i\nu = 2i \\ m = 0 \end{matrix}}$ y_p is of the form: $y_{p_{II}}(t) = A_0 \cos(2t) + B_0 \sin(2t)$

then

$y'_{p_{II}} = -2A_0 \sin(2t) + 2B_0 \cos(2t)$

$y''_{p_{II}} = -4A_0 \cos(2t) - 4B_0 \sin(2t)$

We need

$$\begin{aligned} y''_{p_{II}} + y_{p_{II}} &= -4A_0 \cos(2t) - 4B_0 \sin(2t) + A_0 \cos(2t) + B_0 \sin(2t) \\ &= (-4A_0 + A_0)\cos(2t) + (-4B_0 + B_0)\sin(2t) \\ &= (-3A_0)\cos(2t) + (-3B_0)\sin(2t) = \cos(2t). \end{aligned}$$

then $-3A_0 = 1$, $-3B_0 = 0$

then $A_0 = -\frac{1}{3}$, $B_0 = 0$

$y_{p_{II}}(t) = -\frac{1}{3}\cos(2t)$

2.

$$y'' + y = \sin(t) + te^t$$

$\boxed{y_H}$ $y''+y=0$

$p(z)=z^2+1$, with roots $\pm i$

then $y_H(t) = c_1\cos(t) + c_2\sin(t)$

$\boxed{y_p}$ To find a particular solution split into two problems

$\boxed{y_p \mid I}$ $y''+y=\sin(t)$ We have $L(e^{it}) = \cancel{p(i)}^{0} e^{it}$

$d=0$

$\mu+i\nu=i$

$m=1$

$L(te^{it}) = p'(i)e^{it} + \cancel{p(i)}^{0} te^{it}$

$= 2ie^{it}$ (because $p'(z)=2z$)

then $L\left(\frac{te^{it}}{2i}\right) = e^{it}$, then $y_{pI}(t) = \text{Im}\left(\frac{te^{it}}{2i}\right)$

$\frac{te^{it}}{2i} = \frac{t(\cos(t)+i\sin(t))}{2i}$, then $\text{Im}\left(\frac{te^{it}}{2i}\right)$

$= \text{Im}\left(\frac{t\cos(t)}{2i}\right) = \text{Im}\left(\frac{2it\cos(t)}{-4}\right) = -\frac{1}{2}t\cos(t)$

then $y_{pI}(t) = -\frac{1}{2}t\cos(t)$

$\boxed{y_p \mid II}$ $y''+y=te^t$

$d=0$

$\mu+i\nu=1$

$m=0$

$L(e^t) = p(1)e^t = 2e^t$

$L(te^t) = p'(1)e^t + p(1)te^t = p'(1)e^t + 2te^t = 2e^t + 2te^t$

then $L\left(\frac{te^t - e^t}{2}\right) = te^t$ and then $y_{pII}(t) = \frac{te^t - e^t}{2}$

$\boxed{\text{General Solution}}$

$y(t) = y_H(t) + y_{pI}(t) + y_{pII}(t) = c_1\cos(t) + c_2\sin(t) - \frac{1}{2}t\cos(t) + \frac{te^t - e^t}{2}$.

2.

$$y'' + y = \sin(t) + te^t$$

Alternatively, find the particular solutions by the method of undetermined coefficients.

$\boxed{y_p\,I}$ $y''+y=\sin(t)$

$d=0$
$\mu+i\nu=i$
$m=1$

then y_{pI} is of the form:

$$y_{pI} = A_0 t\cos(t) + B_0 t\sin(t).$$

$$y'_{pI} = A_0\cos(t) - A_0 t\sin(t) + B_0\sin(t) + B_0 t\cos(t)$$

$$y''_{pI} = -A_0\sin(t) - A_0\sin(t) - A_0 t\cos(t) + B_0\cos(t) + B_0\cos(t) - B_0 t\sin(t)$$

we need $y''_{pI} + y_{pI} = \sin(t)$

$$y''_{pI} + y_{pI} = A_0 \cancel{t\cos(t)} + B_0 \cancel{t\sin(t)} - A_0\sin(t) - A_0\sin(t) - A_0 \cancel{t\cos(t)} + B_0\cos(t) + B_0\cos(t) - B_0 \cancel{t\sin(t)}$$

$= 2B_0\cos(t) - 2A_0\sin(t) = \sin(t)$, then $B_0 = 0$, $A_0 = -\frac{1}{2}$

$$y_{pI}(t) = -\frac{1}{2}t\cos(t)$$

$\boxed{y_p\,II}$ $y''+y=te^t$

$d=1$
$\mu+i\nu=1$
$m=0$

$$y_{pII}(t) = (A_0 t + A_1)e^t = A_0 te^t + A_1 e^t$$
$$y'_{pII}(t) = A_0 e^t + A_0 te^t + A_1 e^t$$
$$y''_{pII}(t) = A_0 e^t + A_0 e^t + A_0 te^t + A_1 e^t$$
$$= 2A_0 e^t + A_0 te^t + A_1 e^t$$

Need $y''_{pII} + y_{pII} = te^t$

$$y''_{pII} + y_{pII} = 2A_0 e^t + A_0 te^t + A_1 e^t + A_0 te^t + A_1 e^t = (2A_0 + 2A_1)e^t + (2A_0)te^t$$

then
$2A_0 = 1$
$2A_0 + 2A_1 = 0$

$$\Rightarrow A_0 = \frac{1}{2},\ A_1 = -\frac{1}{2}$$

$$y_{pII}(t) = \frac{1}{2}te^t - \frac{1}{2}e^t$$