MATH246 Summer II
QUIZ 8

Name:

Consider the nonlinear planar system

$$x' = -3x + y\,, \qquad y' = -2x - y - 5x^2\,.$$

1. This system has two stationary solutions. Find them.
2. Find the linearization about each stationary solution.
3. Classify the type of each linearization.
4. Sketch the phase-plane portrait of the system near each stationary solution.

1. Need to solve

$$0 = f(x,y) = -3x + y \qquad \text{(I)}$$
$$0 = g(x,y) = -2x - y - 5x^2 \qquad \text{(II)}$$

From (I): $y = 3x$

substitute into (II): $-2x - 3x - 5x^2 = 0$

$$-5x - 5x^2 = 0$$
$$5x(1+x) = 0$$

then $x = 0$ or $x = -1$

if $x = 0$, then from (I): $y = 0$

if $x = -1$, then from (II): $-3(-1) + y = 0$

$$3 + y = 0$$
$$y = -3$$

The stationary solutions are $(0,0)$, $(-1,-3)$.

2. We have

$$\frac{\partial f}{\partial x} = -3 \qquad \frac{\partial f}{\partial y} = 1$$
$$\frac{\partial g}{\partial x} = -2 - 10x \qquad \frac{\partial g}{\partial y} = -1$$

then about $(0,0)$:

$$\frac{d}{dt}\begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} = \begin{pmatrix} -3 & 1 \\ -2 & -1 \end{pmatrix} \begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix}$$

And about $(-1,-3)$:

$$\frac{d}{dt}\begin{pmatrix}\hat{x}\\ \hat{y}\end{pmatrix} = \begin{pmatrix}-3 & 1\\ 8 & -1\end{pmatrix}\begin{pmatrix}\hat{x}\\ \hat{y}\end{pmatrix}$$

3.

For $(0,0)$:

$$A = \begin{pmatrix}-3 & 1\\ -2 & -1\end{pmatrix}$$

then $p_A(z) = \det(zI - A)$

$$= \det\begin{pmatrix}z+3 & -1\\ 2 & z+1\end{pmatrix} = (z+3)(z+1 \ldots$$
$$= z^2+4z+ \ldots$$
$$= (z+2)^2 + 1 \ldots$$

then $\lambda_1 = -2+i$
$\lambda_2 = -2-i$

this system is a spiral sink (spiral because … eigenvalues are … and a sink because the real part of … complex conjugate pair is …

For $(-1,-3)$

$$A = \begin{pmatrix}-3 & 1\\ 8 & -1\end{pmatrix} \quad p_A(z) = \det(zI - A) = \det\begin{pmatrix}z+3 & -1\\ -8 & z+\ldots\end{pmatrix}$$

$$= (z+3)(z+1) - 8 = z^2+4z-5 = (z+\ldots$$

then $\lambda_1 = -5$, $\lambda_2 = 1$

The associated eigenvalues can be found using:

$$(A-\lambda_1 I)(A-\lambda_2 I) = 0 = (A-\lambda_2 I)(A-\lambda_1 I)$$

$$(A-\lambda_1 I)(A-\lambda_2 I) = \begin{pmatrix} 2 & 1 \\ 8 & 4 \end{pmatrix}\begin{pmatrix} -4 & 1 \\ 8 & -2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

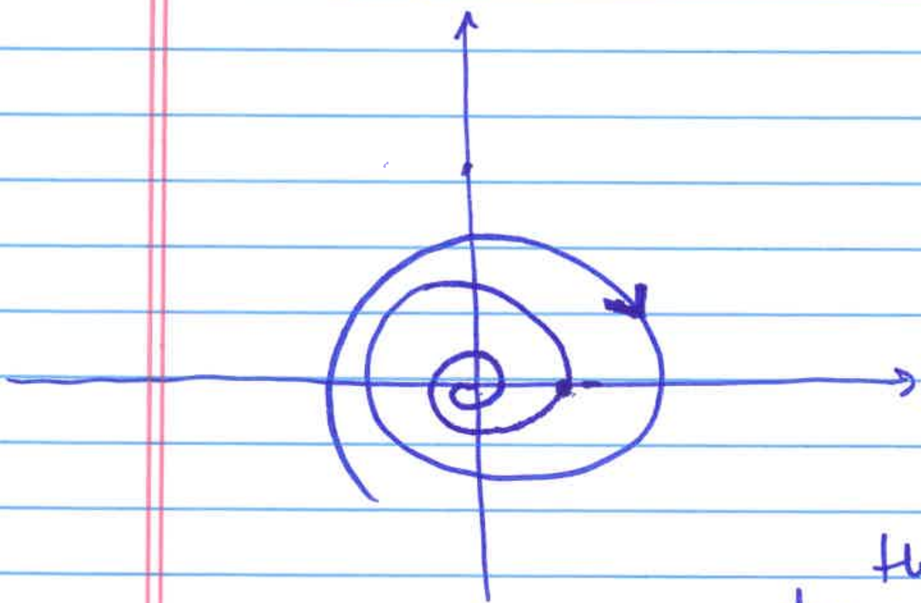
then $\left(-5, \begin{pmatrix} 1 \\ -2 \end{pmatrix}\right)$ is an eigenpair

$$(A-\lambda_2 I)(A-\lambda_1 I) = \begin{pmatrix} -4 & 1 \\ 8 & -2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 8 & 4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

then $\left(1, \begin{pmatrix} 1 \\ 4 \end{pmatrix}\right)$ is an eigenpair.

The linear system correspond to a saddle. (two di
eigenva

4. For $(0,0)$:



Note that
velocity a
is
$$\begin{pmatrix} -3 & 1 \\ -2 & -1 \end{pmatrix}($$
then the spiral
traversed clockwise.

For $(-1,-3)$:

(eigenvalue -5)

The stable line is defined by the vector $\begin{pmatrix}1\\-2\end{pmatrix}$

the unstable line is defined by the vector $\begin{pmatrix}1\\4\end{pmatrix}$

(eigenvalue 1).

then: